Roll No...................

BCA-303

B.C.A. (Semester III) Examination – 2011
Paper: Third
Mathematical Foundations of Computer Science-III

Time: Three Hours] *[Maximum Marks:* **75**
[Minimum Pass Marks: **26**

Note: Section A is compulsory. Attempt any seven questions from Section B and one question from Section C.

Section-A
(Numerical/Analytical/Problematic Questions)

1. (a) Find the rank of the matrix **(4)**

$$\begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

(b) Test the convergence of the series whose n^{th} term is: **(3)**

$$U_n = \sqrt{n^3 + 1} - \sqrt{n^3}$$

2. (a) If $u = x + 2y + z, v = x - 2y + 3z,$ **(4)**

$w = 2xy - zx + 4yz - 2z^2$ Show that:

$J(u, v, w) = 0$

(b) Find maximum or minimum value of u where: $u = xy + \frac{a^3}{x} + \frac{a^3}{y}$ **(4)**

Section- B (6 marks each)

(Short Answer Type Questions)

3. Determine the nature of the series:

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \cdots$$

4. Test the convergence or divergence of the series whose general terms: $U_n = \frac{logn}{n}, n \geq 2$

5. Using Maclaurins expansion shows that:

$$\log_e(1 + \sin^2 x) = \frac{x^2}{1} - \frac{5}{6} x^4 + \cdots$$

6. If $u = (x^2 + y^2 + z^2)^{-1/2}$ show that

$$x \frac{\partial y}{\partial x} + y \frac{\partial y}{\partial x} + z \frac{\partial y}{\partial z} = -y$$

7. If $u = \log_e(x^2 + y^2 + z^2)$ show that:

$$x \frac{\partial^2 y}{\partial y\, \partial x} = y \frac{\partial^2 y}{\partial z\, \partial x}$$

8. If J is the Jacobian of the system x, y with respect to θand ∅ and J′ is the Jacobian of θ, ∅ with respect to x and y then prove that JJ'=1.

9. If $u^3 + v^3 + w^3 = x + y + z$

$u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and

$u + v + w = x^2 + y^2 + z^2$ then show that

$$\frac{\partial(u,v,w)}{\partial\,(x,y,z)} = \frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$$

10. Discuss the maximum and minimum of the function

$u = x^3 y^2 (1 - x - y)$

11. Find the characteristics roots (or eigen values) of the matrix:

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

12. Check if the system of equation has a solution. If yes find the solution using Cramer's rule.

$$3x + y - z = 2$$
$$x + 2y + z = 3$$
$$-x + y + 4z = 9$$

Section –C **(18 marks)**

(Long Answer Type Questions)

13. State and prove that Euler's theorem for Homogeneous functions of degree n. Using Euler's theorem prove that if $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

14. State and prove Cayley-Hamilton theorem; also verify the theorem for the matrix: $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

15. Using Lagrange's method of undetermined multipliers find the maximum value of $u = xyz$ Subject to the condition:

$x + y + z = 8$