BCA – 205(N)

B. C. A. (Second Semester) EXAMINATION, May, 2012

(New Scheme)

Paper Fifth

MATHEMATICS – II

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions from Section B and *one* question from Section C.

Section – A 15

(Numerical/Analytical/Problematic Questions)

1. (a) Find the equation of the sphere whose center is $(2, -3, 4)$ and which passes through the point $(1, 2, -1)$.

   (b) If $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$, then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

2. (a) Evaluate :

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\,dy}{1+x^2+y^2}$$

   (b) Draw the Hasse diagram for the partial ordering $\{(A, B) : A \subseteq B\}$ on the power set P (s), where $S = \{a, b, c\}$.

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Section – B

(Short Answer Type Questions)

3. Change the order of integration in the double integral $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\, dx\, dy$ and hence evaluate.

4. If $Z = f(x, y)$, where $x = e^4 \cos v$ and $y = e^4 \sin v$, show that :

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}$$

5. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact.

6. Evaluate :

$$\int_{-c}^{c}\int_{-b}^{-b}\int_{-a}^{a}(x^2 + y^2 + z^2)\, dx\, dy\, dz.$$

7. In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea :
   (i) How many drink tea and coffee both ?
   (ii) How many drink coffee but not tea ?

8. Prove the following :
   (i) $(A \cap B)' = A' \cup B'$
   (ii) $(A \cup B)' = A' \cap B'$

9. Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$, $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Find also its equations and the points in which it meets the given lines.

10. Use distributive laws to prove the following :

    (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

    (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

11. Prove that mapping $f : N \rightarrow N$ defined by $f(n) = 2n + 3$ where $n \in N$ is one-one and onto.

12. A variable plane is at a constant distance p from the origin and meets the co-ordinate axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is :

$$x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$$

Section – C 15

(Long Answer Type Questions)

13. State and prove the Euler's theorem for homogeneous function of degree n. Using Euler theorem prove that if $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ then :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

14. Discuss the maximum or minimum values of :

$$u = x^3 + y^3 - 3axy$$

15. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find their point of intersection. Also find the equation of the plane in which they lie.