Roll No. .....

BCA–405(N)

B. C. A. (Fourth Semester) EXAMINATION, May, 2013

(New Course)

Paper Fifth

MATHEMATICS – III

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *ten* questions out of thirteen questions from Section B and *one* question from Section C.

Section – A

1. Let $z_1 = 8 + 3i$ and $z_2 = 9 - 2i$. Find : 10

   (i) $z_1 - z_2$

   (ii) $z_1 \cdot \bar{z}_2$

   (iii) z_1^2

   (iv) $\dfrac{1}{z_2}$

2. Which of the following sequences are convergent ? Find the limit of each convergent sequence : 10

   (i) $a_n = \dfrac{1 + (-1)^n}{n}$

   (ii) $a_n = \sqrt{n}$

   where a_n denotes the nth term of sequence.

Section – B

3. Find all roots of $(1 + i)^{1/3}$. 4

4. Find Polar form of $-2 + 2i$. 4

5. Solve : 4

$$\frac{dy}{dx} + \cot y = 0$$

6. Solve : 4

$$3\frac{dy}{dx} + 3\frac{y}{x} = 2x^4 y^4$$

7. Solve : 4

$$(y^4 + 2y)\,dx + (xy^3 + 2y^4 - 4x)\,dx = 0$$

8. Show that $\text{div}\,(\text{grad}\,r^n) = n\,(n+1)\,r^{n-2}$ where $r = \sqrt{x^2 + y^2 + z^2}$. 4

9. Find directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point (1, 1, 1) in the direction of $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. 4

10. Solve the differential equation : 4

$$(D^2 - 2D + 2)y = e^x \cos x$$

11. Using method of undetermined coefficients find the solution of $(D^2 + 4)\,y = \sin 3x$. 4

12. Find half range sine series expansion of function $\pi x - x^2$ in the interval $(0, \pi)$. 4

13. Find the solution of $x^2y'' - 2xy' - 4y = x^4$. 4

14. Find all solutions of $e^z = 1$, where $z = x + iy$. 4

15. Test for the convergence of series : 4

$$\frac{x}{1\cdot 2} + \frac{x^2}{3\cdot 4} + \frac{x^3}{5\cdot 6} + \ldots\ldots$$

Section – C

16. Define Irrotational motion. A fluid motion is given by $\vec{V} = (y \sin z - \sin x)\,\hat{i} + (x \sin z + 2yz)\,\hat{j} + (xy \cos z + y^2)\,\hat{k}$. Is the motion irrotational ? If so, find velocity potential. 15

17. Obtain Fourier series for the function $f(x) = x^2$, $-\pi \le x \le \pi$. Hence show that : 15

(i) $\sum_1^\infty \frac{1}{n^2} = \frac{\pi^2}{6}$

(ii) $\sum_1^\infty \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

(iii) $\sum_1^\infty \frac{(-1)^{n+1}}{n} = \frac{\pi^2}{12}$

18. Which of the following series are convergent and which are divergent ? Give reason : 15

(i) $\sum_2^\infty \frac{1}{n \ln n}$

(ii) $\sum_1^\infty \frac{1 + \cos n}{n^2}$

(iii) $\sum_1^\infty \frac{(2n)!}{(n!)\,(n!)}$