Roll No. ....

BCA–303(O)

B. C. A. (Third Semester) EXAMINATION, Dec., 2012

(Old Course)

Paper Third

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE – III

Time : Three Hours]

[*Maximum Marks : 75*

[*Minimum Pass Marks : 26*

Note : Section A is compulsory. Attempt *seven* questions out of ten from Section B and *one* question from Section C.

Section – A

(Numerical/Analytical/Problematic Questions)

1. (a) Find rank of the matrix : 4

$$\begin{bmatrix} 6 & 8 & 10 \\ 2 & 4 & 6 \\ 4 & 6 & 8 \end{bmatrix}$$

(b) Test the convergence of the series whose *n*th term is : 3

$$U_n = \sqrt{n^3+1} - \sqrt{n^3-1}$$

2. (a) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_1 x_3}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$, prove that : 4

$$J(y_1, y_2, y_3) = 4$$

(b) Find the maximum/minimum values of : 4

$$u = x^3 y^2 (1 - x - y)$$

Section – B 6 each

(Short Answer Type Questions)

3. Determine the nature of the series :

$$1 + \frac{2^2}{3^2} + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2} + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2} + \ldots\ldots$$

4. Test the convergence or divergence of the series whose general term is :

$$U_n = \frac{\log n}{n}, n \geq 2$$

5. Expand $e^{a \sin^{-1} x}$ by Maclaurin's theorem and find the general term.

6. If $u = x^2 + y^2 + z^2$ show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 2u$$

7. If $u = e^{xyz}$ prove that :

$$\frac{\partial^3 u}{\partial x \, \partial y \, \partial z} = (1 + 3xyz + x^2 y^2 z^2)\, e^{xyz}$$

8. If J is the Jacobian of the system x, y with respect to θ and ϕ and J′ is the Jacobian of θ, ϕ with respect to x and y, then prove that :

$$J\,J' = 1$$

9. If $u^3 + v^3 = x + y$ and $u^2 + v^2 = x^3 + y^3$, then show that :

$$\frac{\partial (u, v)}{\partial (x, y)} = \frac{1}{2} \cdot \left[\frac{y^2 - x^2}{uv\,(u - v)}\right]$$

10. Discuss maximum and minimum of :

$$u = xy(1 - x - y)$$

11. Find the characteristic roots (or eigen values) of the matrix :

$$\begin{bmatrix} 2 & -4 & 3 \\ -6 & 7 & -4 \\ 8 & -6 & 2 \end{bmatrix}$$

12. Check if the system of equations has a solution. If yes, find the solution using Cramer's rule :

$$2x + 4y + 2z = 6$$

$$6x + 2y - 2z = 4$$

$$-2x + 2y + 8z = 18$$

Section – C 18

(Long Answer Type Questions)

13. State and prove the Euler's theorem for Homogeneous functions of degree n. Using Euler theorem prove that if :

$$u = \tan^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$$

then prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

Also find :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

14. State and prove Cayley-Hamilton theorem, also verify the theorem for the matrix :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

15. Using Langrange's method of undetermined multipliers find the minimum value of :

$$u = x^2 + y^2 + z^2$$

subject to the condition :

$$ax + by + cz = p$$

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