Roll No. ....

BCA–205(N)

B. C. A. (Second Semester) EXAMINATION, May, 2013

(New Course)

Paper Fifth

MATHEMATICS – II

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions from Section B and *one* question from Section C.

Section – A

1. (a) Find the equation of the sphere whose center $(-3, 4, 5)$ and radius 7. 4

   (b) If $f(x) = x^3 - 1/x^3$, find the value of $f(x) + f(1/x)$. $3\frac{1}{2}$

2. (a) Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation "*a* divides *b*". Draw the Hasse diagram. 4

   (b) Prove that : $3\frac{1}{2}$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Section – B 45

3. Discuss the maxima and minima of $u = xy(1 - x - y)$.

4. If :

$$u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)\ ;\ xy \neq 0$$

Prove that :

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$$

5. Change the order of integration in the double integral $\int_0^\infty \int_0^x e^{-x^2/y}\, dx\, dy$ and evaluate.

6. Find the equation of the plane through the points (0, 0, 2), (1, 2, 1) and (3, 1, 0).

7. Evaluate the integral :

$$\int_0^{\log 2} \int_0^x \int_0^{x + \log y} e^{x+y+z}\, dz\, dy\, dx$$

8. Find the equation of the spheres which passes through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0$, $2x + y + z = 4$ and touch the plane $3x + 4y - 14 = 0$.

9. Evaluate :

$$\int_0^3 \int_1^2 xy\,(1 + x + y)\, dx\, dy$$

10. Find the point on the line :

$$\frac{x-6}{3} = -(y-7) = z - 4 \text{ and } -\frac{x}{3} = \frac{y+9}{2} = \frac{z-2}{4}$$

which are nearest to each other. Hence find the shortest distance between the lines and also its equations.

11. If $u = f(y - z, z - x, x - y)$, prove that :

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

12. If two relations R and S are given by R {(1, 3), (2, 1), (3, 4), (4, 2)} and S = {(1, 2), (2, 3), (3, 4), (4, 1)} find SoR.

Section – C 15

13. State and prove Euler's theorem and give a suitable example.

14. For any sets A and B, prove that :

(i) $A - B = A \cap B'$

(ii) $(A - B) \cup B = A \cup B$

(iii) $(A - B) \cap B = \phi$

15. If $f : X \rightarrow Y$ and A, B are two subsets of Y, then prove that :

$$f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$$

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$