Roll No. ....

BCA–105(N)

B. C. A. (First Semester) EXAMINATION, Dec., 2012

(New Course)

Paper Fifth

MATHEMATICS – I

Time : Three Hours]   [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions out of ten from Section B and *one* question from Section C.

Section – A

(Numerical/Analytical/Problematic Questions)

1. (a) Show that :   4

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2\,abc\,(a+b+c)^3$$

   (b) If $f(x) = (x-1)(x-2)(x-3)$ and $a = 0$, $b = 4$, find c using Lagrange's mean value theorem.   4

2. (a) If $\vec{a} = 2i - 3j - k$ and $\vec{b} = i + 4j - 2k$, find $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$.   3

   (b) Evaluate the following integral :   4

$$\int \frac{x + \sin x}{1 + \cos x}\,dx$$

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Section – B   6 each

(Short Answer Type Questions)

3. Show that $\lim\limits_{x \to 2} \dfrac{|x-2|}{x-2}$ does not exist.

4. Reduce the following matrix into normal form and hence find its rank :

$$A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix}$$

5. Evaluate :

$$\lim\limits_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x^2}$$

6. If :

$$\phi(n) = \int_0^{\pi/4} \tan^n x \, dx$$

then prove that :

$$\phi(n) + \phi(n-2) = \frac{1}{n-1}$$

7. Show that :

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$$

8. Find the volume of the parallelopiped whose edges are represented by vectors $\vec{a} = 2i - 3j$, $\vec{b} = i + j - k$ and $\vec{c} = 3i - k$.

9. Prove that :

$$\int_0^1 x^2 (1-x^2)^{3/2} \, dx = \frac{\pi}{32}$$

10. Show that the function defined by :

$$f(x) = \begin{cases} 5x - 4, & \text{if } 0 < x \le 1 \\ 4x^3 - 3x, & \text{if } 1 < x < 2 \end{cases}$$

is continuous at $x = 1$.

11. Describe different kinds of discontinuities of a function by giving *one* example of each.

12. If :

$$y = \frac{\sin^{-1} x}{\sqrt{1 - x^2}}$$

show that :

$$(1 - x^2) \frac{dy}{dx} = xy + 1$$

Section — C   18 each

(Long Answer Type Questions)

13. Find the eigen values and eigen vectors for the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

14. If :

$$y = (\sin^{-1} x)^2$$

prove that :

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$$

15. Evaluate $\int_a^b x^2 \, dx$, directly from the definition of integral as the limit of a sum.

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