Roll No. ....

BCA–405(N)

B. C. A. (Fourth Semester)
EXAMINATION, May/June, 2015

(New Course)

Paper Fifth

MATHEMATICS—III

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *two* questions from Section B and *two* questions from Section C.

Section—A 3 each

(Short Answer Type Questions)

1. (A) Find all value of $(1+i)^{1/3}$.

(B) Find the values of constants 'a', 'b' and 'c' if :

$$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$$ is irrotational.

(C) Find the value of $\log(1+i)$ in the form of $a + ib$.

(D) Examine the convergence of series :

$$\frac{1}{3.7} + \frac{1}{4.9} + \frac{1}{5.11} + \frac{1}{6.13} + \ldots\ldots$$

(E) Solve :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y} .$$

(F) Expand $f(x) = (x - 1)$ as half range sine series in $0 < x < 2$.

(G) Find the solution of $\left(x^2 + y^2 + 1\right) dx - 2\,xy\,dy = 0$.

(H) Discuss the nature of series $\sum_{1}^{\infty} \frac{x^{2n-1}}{\sqrt{2n+1}}$.

(I) Find the value of $(z_1 + z_2)/z_3$ where $z_1 = 4 - 3i,\ z_2 = 2 - i$ and $z_3 = 1 - i$.

Section—B 12 each

(Long Answer Type Questions)

2. Find Fourier series for function $f(x)$ in interval $(-1, 1)$ where $f(x) = \begin{cases} x + 1 & -1 < x < 0 \\ x - 1 & 0 < x < 1 \end{cases}$. Hence deduce that :

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

3. Find the complete solution of :

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$$

4. Expand $f(x) = x^2, -\pi < x < \pi, f(x + 2\pi) = f(x)$ in Fourier series. Hence deduce that :

$$\frac{\pi^2}{12} = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} \ldots\ldots$$

5. Solve the differential equation :

(i) $2\frac{dy}{dx} - y \sec x = y^3 \tan x$

(ii) $\frac{d^2y}{dx^2} - 4y = \cos^2 x$

Section—C 12 each

(Long Answer Type Questions)

6. Show that :

(i) $\text{div}\left(r^n \vec{r}\right) = (n + 3)\, r^n$

(ii) $\text{curl}\left(r^n \vec{r}\right) = 0$

where $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ and $r = \left|\vec{r}\right|$.

7. Examine the convergence of series :

$$1 + \frac{2x}{2!} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \ldots\ldots\ldots, (x > 0)$$

8. (i) Find the unit tangent vector at any point on the curve $x = t^2 + 2,\ y = 4t - 5,\ z = 2t^2 - 6t$ where t is any variable.

(ii) Find directional derivative of $\phi = x\,y\,z^2$ at point $(1, 0, 3)$ in the direction of $2\hat{i} + 3\hat{j} + 6\hat{k}$.

9. (a) Determine the region of Argand diagram defined by $|z - 1| \le 2$.

(b) Solve :

$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$