Section—C

18 each

(Long Answer Type Questions)

13. By using Maclaurin's theorem find the first four terms in the expansion of $\log(1+\tan x)$ in the power of x.

14. If xyz are all different and if :

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

prove that $xyz = -1$.

15. Show that the matrix :

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

satisfies its own characteristic equation. Hence find A^{-1}.

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Roll No. ....

BCA–105(N)

B. C. A. (First Semester) EXAMINATION, Dec., 2014

(New Course)

Paper Fifth

MATHEMATICS—I

Time : Three Hours] [*Maximum Marks : 75*

Note : Section A is compulsory. Attempt any *seven* questions out of ten from Section B and any *one* question from Section C.

Section—A

1. (a) Using Crammer's rule solve the following system of equations : 4

$$2x - 3y + z = 7$$

$$2x + y - z = 1$$

$$4y + 3z = -11$$

(b) Find the value of : 4

$$\int \frac{\sin^4 x}{\cos^2 x}\, dx$$

2. (a) Evaluate : 2 each

(i) $\lim_{x\to 0} \dfrac{(1+x)^6 - 1}{(1+x)^2 - 1}$

(ii) $\lim_{x\to 0} = \dfrac{e^x - 2\cos x + e^{-x}}{x^2}$

(b) Find C of the mean value theorem when : 3

$$f(x) = x^3 - 3x - 2$$

in $[-2, 3]$.

Section—B

(Short Answer Type Questions)

Note : Attempt any *seven* questions. Each question carries 6 marks.

3. Show that :

$$\begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

4. Find the rank of matrix :

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 2 & 5 & 1 \\ 0 & 4 & 4 & -4 \end{bmatrix}$$

5. Determine the eigen values and the corresponding eigen vector of the matrix :

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}$$

6. Find the maximum and minimum values of the function :

$$x^3 - 2x^2 + 6$$

7. If :

$$y = x^2 e^x$$

then by Leibnitz theorem prove that :

$$y_n = \frac{1}{2} n(n-1)\frac{d^2 y}{dx^2} - n(n-2)\frac{dy}{dx} + \frac{1}{2}(n-1)(n-2)\, y$$

8. Expand $\log\sin(x+h)$ in power of h by Taylor's theorem.

9. Show that the function defined by :

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

is continuous at $x = 3$.

10. Differentiate the following functions w. r. to x :

(i) $\dfrac{\sin x - x\cos x}{x\sin x + \cos x}$

(ii) $(\log x)^{\sin x}$

11. If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, find the value of $\left(\vec{a} + 3\vec{b}\right) \cdot \left(2\vec{a} - \vec{b}\right)$.

12. Evaluate :

$$\int_0^1 x^4 \left(1 - \sqrt{x}\right)^5 dx$$