

BCA-104(O)

B. C. A. (First Semester) EXAMINATION, Dec., 2014

(Old Course)

Paper Fourth

MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE—I

Time : Three Hours]

[Maximum Marks : 75

Note : Section A is compulsory. Attempt any *seven* questions out of ten questions from Section B and *one* question from Section C.

Section—A

(Numerical/Analytical/Problematic Questions)

1. Discuss the continuity at $x = a$ of the function $f(x)$ given by : 9

$$f(x) = \begin{cases} (x^2/a) - a & 0 < x < a \\ 0 & x = 0 \\ a - (a^3/x^2) & x > a \end{cases}$$

2. Evaluate the following : 5, 4

(i) $\int e^{ax} \sin (bx + c) dx$

(ii) $\int \{\sin (\log x) + \cos (\log x)\} dx$

Section—B

(Short Answer Type Questions)

3. Show that : 6
 $A - (B \cap C) = (A - B) \cap (A - C)$
4. If $f : Z \rightarrow Z$ and $g : Z \rightarrow Z$ are functions and defined by

$$f(x) = x^2 + 2x - 1$$

$$g(x) = 2x - 1,$$

then find :

3, 3

(a) $f \cdot g$

(b) $g \cdot g$

5. If $y = (x^2 - 1)^n$, prove that :

6

$$(x^2 - 1) y_{n+2} + 2x y_{n+1} - n(n+1) y_n = 0$$

6. Show that :

6

$$\lim_{x \rightarrow a} \left[\frac{x e^{-x} - a e^{-a}}{x - a} \right] = (1 - a) e^{-a}$$

7. Evaluate :

6

$$\int \frac{d\theta}{\sin^4 \theta/2}$$

8. If $A = \{1, 2\}$ and $B = \{3, 4\}$, then find :

3, 3

(i) $(A \times B) \cup (B \times A)$

(ii) $(A \times B) \cap (B \times A) \cap B$

9. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ are two functions, then prove that, if f and g are surjective, then $g \cdot f$ is also surjective. 6

10. Prove the given identity : 6

$$A \cup (B \cap C) = (A \cup B) \cap C$$

also represent it using Venn diagram.

11. Find the area of a loop of the curve : 6

$$r = a \sin 3\theta$$

12. Find y_n , when :

6

$$y = e^x \log x$$

Section—C

(Long Answer Type Questions)

13. Change the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} \sin nx \, dx \, dy$, show that : 15

$$\int_0^\infty \frac{\sin nx}{x} \, dx = \frac{\pi}{2}$$

14. (a) Prove that :

8

$$\tan^{-1}(x+h) = \tan^{-1} x + h \sin z \left(\frac{\sin z}{1} \right)$$

$$-(h \sin z)^2 \frac{\sin 2z}{2} + (h \sin z)^3 \frac{\sin 3z}{3} + \dots$$

- (b) If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that :

7

$$P + \frac{d^2 P}{d\theta^2} = \frac{a^2 b^2}{P^3}$$