

Roll No.

BCA-105(N)

B. C. A. (First Semester)

EXAMINATION, Dec., 2013

(New Course)

Paper Fifth

MATHEMATICS-I

Time : Three Hours] [Maximum Marks : 75

Note : Section A is compulsory. Attempt any seven questions out of ten from Section B and any one question from Section C.

Section—A

(Numerical/Analytical/Problematic Questions)

1. (a) Verify Cayley-Hamilton theorem for the matrix : 4

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

(b) A function $f(x)$ is defined as follows : 4

$$f(x) = \begin{cases} (x^2/a) - a & \text{when } x < a \\ 0 & \text{when } x = 0 \\ a - (x^2/a) & \text{when } x > a \end{cases}$$

Prove that the function $f(x)$ is continuous at $x = a$.

2. (a) Find the maximum and minimum values of the function $y = 2x^3 - 9x^2 + 12x - 1$. 3

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(b) Evaluate :

$$I = \int \frac{dx}{\sin(x-a)\sin(x-b)}$$

Section—B

(Short Answer Type Questions)

Note : Attempt any seven questions. Each question carries 6 marks.

3. Using properties of determinants, solve the following determinant for x :

$$\begin{vmatrix} a+x & a-x & a-x & a-x \\ a-x & a+x & a-x & a-x \\ a-x & a-x & a-x & a+x \end{vmatrix} = 0$$

4. Using Cramer's rule, solve the following system of linear equations :

$$\begin{aligned} x+2y+3z &= 6 \\ 2x+4y+z &= 17 \\ 3x+2y+9z &= 2 \end{aligned}$$

5. Evaluate :

$$\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$$

6. Define various kinds of discontinuity with suitable examples.

7. If :

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

then, show that $f(x)$ is continuous and differentiable everywhere.

8. Verify Rolle's theorem for the function :

$$f(x) = x^3 - 6x^2 + 11x - 6$$

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9. Evaluate :

$$\int \frac{\cos^9 x}{\sin x} dx$$

10. From the definition of a definite integral as the limit of a sum, evaluate $\int_a^b e^x dx$.

11. If a, b, c are three vectors such that $a + b + c = 0$, then show that :

$$a \times b = b \times c = c \times a$$

12. Find the area of triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$.

Section—C

(Long Answer Type Questions)

13. Find the eigen values and the corresponding eigen vectors of the matrix :

$$\begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$

14. If $y = [x + \sqrt{1+x^2}]^m$, find $(y^n)'$.

15. (a) Show that :

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta d\theta = \frac{[\frac{p+1}{2}]! [\frac{q+1}{2}]!}{2^{\frac{p+q+1}{2}}}$$

(b) Find a set of vectors reciprocal to the three given vectors :

$$a = -i + j + k, b = i - j + k \text{ and } c = i + j + k$$

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